the computational procedure. The vertical lines drawn in divide the decimal part of the table into groups of five places.

Values indicated by a check have been recalculated either by another computer or by comparison with previous tabulations.

AUTHOR'S SUMMARY

37[E].—H. C. SPICER, *Tables of the Descending Exponential Function* e^{-x}, U. S. Geological Survey, Washington 25, D. C. Deposited in UMT File.

This manuscript is in the form of original computation sheets. It contains the values for e^{-x} with x ranging in value as follows: [0(0.0001)1] 25D; [1(0.001)3.923] 25D; [3.923(0.01)10] 25D.

On each sheet the column indicated as x, the argument, is followed immediately on the same line with the 25-decimal-place value of e^{-x}. All of the values tabulated between two tabular values of e^{-x} are not to be used, as they were obtained as parts of the computational procedure.

The values indicated by C. K. at each 0.0005 mid-value are check values obtained by an additional computation. The difference between the two values is only indicated for the digits at the end of the value.

The values indicated by T. V. are comparison values from previous tabulations. The difference, as before, is only indicated for the end digits.

AUTHOR'S SUMMARY

38[F].—ROBERT SPIRA, *Tables Related to* $x^2 + y^2$ *and* $x^4 + y^4$. Five large manuscripts deposited in UMT files.

The following three tables have been computed:

1. All representations of $p^k = a_i^2 + b_i^2$, where p is a prime $\equiv 1 \pmod 4$ and $p < 1000$. The k's are such that $\max(a_i, b_i) < 2^{35}$. The factorizations of a_i and b_i are also given.

2. All representations of $n = a^2 + b^2$ for $n < 122{,}500$. Also given are the factorizations of n, a, and b. The table continues to $n = 127{,}493$ but is not complete here, since a and b are always less than 350. Francis L. Miksa [1] has previously given the representations of the odd $N < 100{,}000$; as he explains in his introduction, the even N are easily derived from these. Miksa did not give the factorizations of n. It is not clear why Spira factors a and b also.

3. All representations of $n = a^4 + b^4$ for a and $b \leqq 350$. The table is thus complete for $n < 351^4 = 15{,}178{,}486{,}401$ but continues up to $n = 350^4 + 350^4$. Also given are the factorizations of n, a, and b.

This last table was searched for solutions of

$$U^4 + V^4 = W^4 + T^4,$$

and only the three known solutions, for U, V, W, and $T \leqq 350$, were found. This confirms the result of Leech [2]. The author adds that there is no solution of $U^5 + V^5 = W^5 + T^5$ for U, V, W, and $T \leqq 110$.

The calculations were done using a sorting routine on an IBM 704 in the University of California Computer Center.

D. S.

1. FRANCIS L. MIKSA, *Table of quadratic partitions* $x^2 + y^2 = N$, RMT **83**, *MTAC*, v. 9, 1955, p. 198.
2. JOHN LEECH, "Some solutions of Diophantine equations," *Proc. Cambridge Philos. Soc.*, v. 53, 1957, p. 778–780.

39[F].—DAVID C. MAPES, *Fast Method for Computing the Number of Primes less than a Given Limit*, Lawrence Radiation Laboratory Report UCRL-6920, May 1962, Livermore, California. Table of 20 pages deposited in UMT File.

This report is the original writeup of [1]. The table in [1] gives $\pi(x)$, $Li(x)$, $R(x)$, $L(x) - \pi(x)$ and $R(x) - \pi(x)$ for $x = 10^7(10^7)10^9$, where $\pi(x)$ is the number of primes $\leqq x$, and $Li(x)$ and $R(x)$ are Chebyshev's and Riemann's approximation formulas. The table here gives the same quantities for $x = 10^6(10^6)10^9$. It thus has greater "continuity," but not enough to trace the course of $\pi(x)$ unequivocally.

For example, Rosser and Schoenfeld [2] have recently proved that $\pi(x) < Li(x)$ for $x \leqq 10^8$. While it is highly probable that this inequality continues to $x = 10^9$, the gaps here, of $\Delta x = 10^6$, would appear to preclude a *rigorous* proof at this time. Study of the table, however, shows no value of x for which $\pi(x)$ approaches $Li(x)$ sufficiently close to arouse much suspicion. The relevant function is

$$PI(x) = \frac{Li(x) - \pi(x)}{\sqrt{x}} \log x,$$

and for $313 \leqq x \leqq 10^8$, Appel and Rosser [3] showed a minimum value of $PI(x)$, equal to 0.526, at $x = 30{,}909{,}673$. Here (and also in [1]) one finds values of 0.615 and 0.543 at $x = 110 \cdot 10^6$ and $180 \cdot 10^6$, respectively. It is thus likely that a value of $PI(x)$ less than 0.526 can be found in the neighborhood of these x (especially the second), but it is unlikely that $PI(x)$ becomes negative there. The relevant theory [4] is made difficult by incomplete knowledge of the zeta function. In the second half of the table, $x > 500 \cdot 10^6$, no close approaches at all are noted, and $Li(x) - \pi(x)$ exceeds 1000 there, except for $x = 501 \cdot 10^6$, $604 \cdot 10^6$, and $605 \cdot 10^6$.

The low values of $PI(x)$ are always associated with the condition $\pi(x) > R(x)$. The largest value of $R(x) - \pi(x)$ shown here is $+914$, for $x = 905 \cdot 10^6$.

D. S.

1. DAVID C. MAPES, "Fast method for computing the number of primes less than a given limit, "*Math. Comp.*, v. 17, 1963, p. 179–185.
2. J. BARKLEY ROSSER AND LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, p. 64–94.
3. KENNETH I. APPEL AND J. BARKLEY ROSSER, *Table for Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in RMT **55**, *Math. Comp.*, v. 16, 1962, p. 500–501.
4. A. E. INGHAM, *The Distribution of Prime Numbers*, Cambridge Tract No. 30, Cambridge University Press, 1932.

40[F].—J. BARKLEY ROSSER & LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, Tables I-IV on p. 90–93.

The four number-theoretic tables reviewed here were presented by the authors in connection with their proofs of numerous inequalities concerning the distribution of primes. These inequalities include

$$\frac{x}{\log x}\left(1 + \frac{1}{2 \log x}\right) < \pi(x) < \frac{x}{\log x}\left(1 + \frac{3}{2 \log x}\right) \qquad (59 \leqq x),$$